

Update on the lattice calculation of  $B \rightarrow K^* \gamma$ 

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We present updated results on the calculation of the matrix elements for  $B \rightarrow K^* \gamma$  in the quenched approximation on a  $24^3 \times 48$  lattice at  $\beta=6.2$ , using an  $O(a)$ -improved fermion action. The scaling behaviours of the form factors  $T_1(q^2=0)$  and  $T_2(q_{max}^2)$  for the decay are examined and pole model ansatzes tested.

## 1. Introduction

Theoretical interest in the rare decay  $B \rightarrow K^* \gamma$  as a test of the Standard Model has been renewed by the experimental results of the CLEO collaboration [1]. The viability of calculating the relevant hadronic matrix elements on the lattice was first demonstrated by Bernard, Hsieh and Soni [2] in 1991.

The computational details and results of this work have been described in references [3] and [4].

## 2. Form Factor Definitions

The hadronic matrix elements can be parametrised by three form factors,

$$\langle K^*(k, \epsilon) | \bar{s} \sigma_{\mu\nu} q^\nu b_R | B(p) \rangle = \sum_{i=1}^3 C_\mu^i T_i(q^2), \quad (1)$$

where,

$$C_\mu^1 = 2\varepsilon_{\mu\nu\lambda\rho} \epsilon^\nu p^\lambda k^\rho, \quad (2)$$

$$C_\mu^2 = \epsilon_\mu (m_B^2 - m_{K^*}^2) - \epsilon \cdot q (p+k)_\mu, \quad (3)$$

$$C_\mu^3 = \epsilon \cdot q \left( q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p+k)_\mu \right), \quad (4)$$

and  $q$  is the momentum of the emitted photon.

As the photon emitted is on-shell, the form factors need to be evaluated at  $q^2=0$ . In this limit,

$$T_2(q^2=0) = -iT_1(q^2=0), \quad (5)$$

and the coefficient of  $T_3(q^2=0)$  is zero in the on-shell matrix element. Hence, the branching ratio can be expressed in terms of a single form factor, for example  $T_1(q^2=0)$ .

## 3. Heavy Quark Scaling

We calculate with a selection of quark masses near the charm mass and extrapolate to the  $b$ -quark scale. In the heavy quark limit, heavy quark symmetry [5] tells us that,

$$\begin{aligned} T_1(q_{max}^2) &\sim m_P^{1/2} \\ T_2(q_{max}^2) &\sim m_P^{-1/2}, \end{aligned} \quad (6)$$

where  $m_P$  is the pseudoscalar mass. Combining this with the relation  $T_2(q^2=0) = -iT_1(q^2=0)$  constrains the  $q^2$  dependence of the form factors. However, it does not provide a scaling law for  $T_1(q^2=0)$  without further assumptions about the actual  $q^2$  behaviour of the form factors.

Pole dominance ideas suggest that,

$$T_i(q^2) = \frac{T_i(0)}{(1 - q^2/m_i^2)^{n_i}}, \quad (7)$$

for  $i = 1, 2$ , where  $m_i$  is a mass that is equal to  $m_P$  plus  $1/m_P$  corrections and  $n_i$  is a power. Since  $1 - q_{max}^2/m_i^2 \sim 1/m_P$  for large  $m_P$ , the combination of heavy quark symmetry and the form factor relation at  $q^2=0$  implies that  $n_1 = n_2 + 1$ . For example,  $T_2(q^2)$  could be a constant and  $T_1(q^2)$  a single pole, or  $T_2(q^2)$  could be a single pole and  $T_1(q^2)$  a double pole. These two cases correspond to,

$$T_1(0) \sim \begin{cases} m_P^{-1/2} & \text{single pole} \\ m_P^{-3/2} & \text{double pole} \end{cases}. \quad (8)$$

The data appear visually to favour  $T_2(q^2)$  constant in  $q^2$  when  $m_P$  is around the charm scale. However, we will consider both constant and single pole behaviours for  $T_2(q^2)$  below.

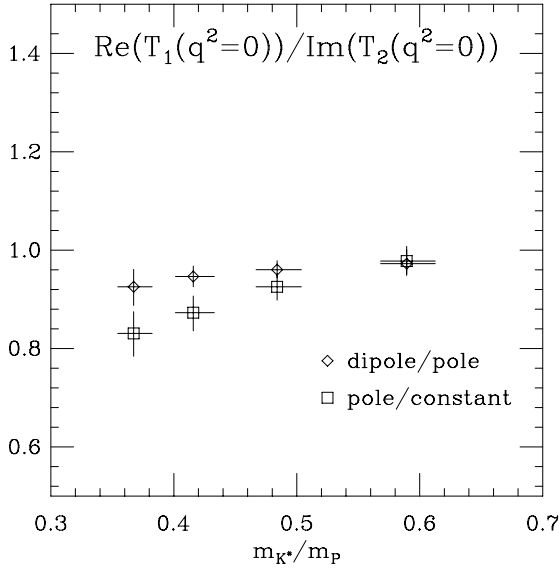


Figure 1. The ratio  $T_1/T_2$  at  $q^2=0$  for dipole/pole and pole/constant fits.

#### 4. Results

As demonstrated in a previous paper [3], the evaluation of  $T_1(q^2; m_P; m_{K^*})$  is relatively straightforward, and  $T_2$  can be determined in a similar way. We fit  $T_1(q^2)$  to a pole or dipole model in order to obtain the on-shell form factor  $T_1(q^2=0)$ ,

$$T_1(q^2) = \frac{T_1(q^2=0)}{1 - q^2/m^2}, \quad \frac{T_1(q^2=0)}{(1 - q^2/m^2)^2}, \quad (9)$$

The difference between the two models was found to be negligible. The form factor  $T_2$  was fitted to a pole model or constant

The ratio  $T_1/T_2$  at  $q^2=0$  for dipole/pole and pole/constant fits is shown in Fig.(1). The magnitude is found to be consistent with 1 at low masses, in accordance with the identity  $T_1(0) = iT_2(0)$ , Eq.(5). At higher masses, the dipole/pole fits for  $T_1/T_2$  deviate less than the pole/constant fits.

#### 5. Extrapolation of $T_2(q_{max}^2)$ to $m_B$

In order to test heavy quark scaling, we also extracted the form factor  $T_2$  at maximum recoil, where  $q^2 = q_{max}^2 = (m_P - m_V)^2$ , in the same way as Bernard *et al.* [6]. In the heavy quark limit,  $T_2(q_{max}^2)$  is expected to scale as  $m_P^{-1/2}$ , analogous to the scaling of  $f_B$ . Higher order  $1/m_P$  and radiative corrections will also be present. For convenience, we remove the leading scaling behaviour by forming the quantity,

$$\hat{T}_2 = T_2(q_{max}^2) \sqrt{\frac{m_P}{m_B}} \left( \frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{2/\beta_0}. \quad (10)$$

The normalisation ensures that  $\hat{T}_2 = T_2(q_{max}^2)$  at the physical mass  $m_B$ . Linear and quadratic correlated fits for  $\hat{T}_2$  were carried out with the functions,

$$\hat{T}_2(m_P) = A \left( 1 + \frac{B}{m_P} \right), \quad (11)$$

$$\hat{T}_2(m_P) = A \left( 1 + \frac{B}{m_P} + \frac{C}{m_P^2} \right), \quad (12)$$

and are shown in Fig.(2). Taking the quadratic fit of  $T_2$  at  $m_P = m_B$  as the best estimate, and the difference between the central values of the linear and quadratic fits as an estimate of the systematic error,  $T_2$  was found to be,

$$T_2(q_{max}^2; m_B; m_{K^*}) = 0.269_{-9}^{+17} \pm 0.011. \quad (13)$$

If the  $q^2$  dependence of  $T_2$  at  $m_B$  were known, this result could be related to  $T_1(q^2=0)$  via the identity  $T_1(0) = iT_2(0)$ .

#### 6. Extrapolation of $T_1(q^2=0)$ to $m_B$

For  $T_1(q^2=0)$  we test the two possible scaling laws in the same way as for  $T_2$ , by forming the quantity,

$$\hat{T}_1 = T_1(q^2=0) \left( \frac{m_P}{m_B} \right)^n \left( \frac{\alpha_s(m_P)}{\alpha_s(m_B)} \right)^{2/\beta_0}, \quad (14)$$

where  $n = 1/2, 3/2$ . For  $n = 3/2$ , a similar scaling relationship has been found using light-cone sum rules by Ali, Braun and Simma [7]. The  $n = 1/2$  case has been suggested by other sum rules calculations [8].

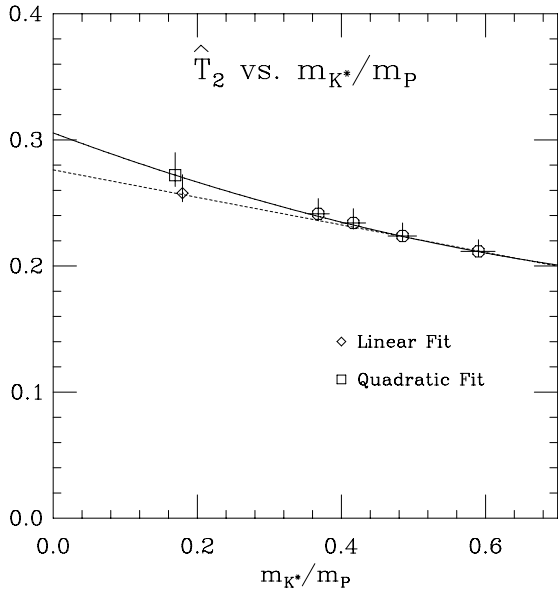


Figure 2.  $\hat{T}_2$  extrapolation, with linear and quadratic fits.

Linear and quadratic fits were carried out with the same functions as for  $T_2$ . The two cases  $n = 1/2, 3/2$  are shown in Fig.(3). The  $\chi^2/\text{d.o.f.}$  are approximately 1 for the scaling laws, indicating that the models are statistically valid in the available mass range.

The final results for  $T_1(q^2=0; m_B; m_{K^*})$  are taken from the quadratic fit for  $T_1$ , with the systematic error estimated as for  $T_2$ ,

$$T_1(q^2=0) = \begin{cases} 0.159_{-33}^{+34} \pm 0.067 & n = 1/2 \\ 0.124_{-18}^{+20} \pm 0.022. & n = 3/2 \end{cases} \quad (15)$$

## 7. Conclusions

Further information on the  $q^2$  dependence of  $T_1$  and  $T_2$  is required to remove the uncertainty in obtaining the form factors at the physical point  $q^2=0, m_P=m_B$ .

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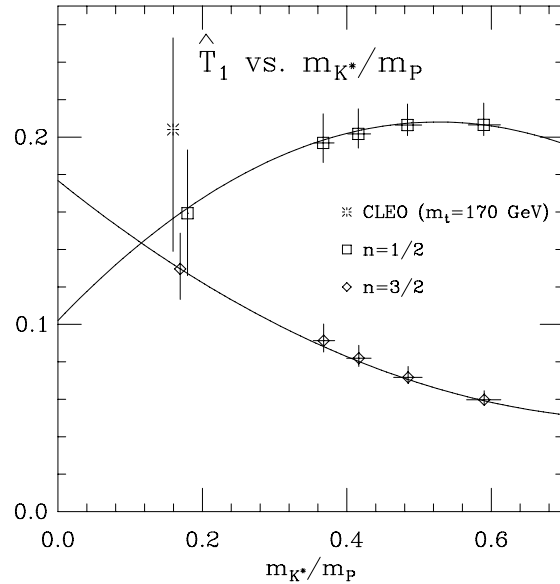


Figure 3.  $\hat{T}_1$  extrapolation, for  $n = 1/2, 3/2$  (Points displaced slightly for clarity).

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